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ABSTRACT

The effect of thickness on the lift and pitching moment on two dimensional pitching airfoils at supersonic speeds is investigated. The airfoils considered have arbitrary symmetrical cross sections, and the flow is supersonic throughout the flow field.

The analysis is based on a second order theory similar to the second order theory introduced by Busemann and extended by Van Dyke. The lifting pressure due to steady pitching is found and this is used to calculate the lift due to pitching, C_{L_q}, and the moment due to pitching, C_{m_q}.

SYMBOLS

c	airfoil chord
C_p	pressure coefficient $\left(\dfrac{\text{Pressure}}{1/2\ \rho\ V^2}\right)$
c_o	velocity of sound in air which has been brought to rest adiabatically
d	distance from airfoil leading edge to axis of pitch
f	arbitrary function associated with the equation of the airfoil surface (see eq. 9)
g_1, g_2, g_3	functions associated with the form of the first order potential function
$h_1, h_2, \ldots h_6$	functions associated with the form of the second order potential function
I_1, I_2	functions associated with the form of the second order lifting pressure
i, k	unit vectors
ℓ	a distance small compared to unity
M	Mach number
q	rate of pitch
s	a function associated with the airfoil surface
V	free stream velocity
t, t_o	time
x, y, z	rectangular coordinates
x_o, y_o, z_o	rectangular coordinates
α	angle of attack
β	$\sqrt{M^2-1}$

γ	adiabatic exponent
ϵ	thickness parameter (see eq. 7)
ξ, η	characteristic coordinates
Φ	second order potential function
ϕ	first order perturbation potential function
ψ	second order perturbation potential function
ψ_1, ψ_2	auxiliary functions used in finding the second order potential function
ω	velocity vector
L	normal force
M'	pitching moment
C_L	force coefficient $= \left(\dfrac{L}{1/2\ \rho\ V^2\ c}\right)$
C_m	pitching moment coefficient $= \left(\dfrac{M'}{1/2\ \rho\ V^2\ c^2}\right)$

$$C_{L_q} = \left. \frac{\partial C_L}{\partial\left(\frac{q\,c}{2\,V}\right)} \right|_{q \to 0}$$

$$C_{m_q} = \left. \frac{\partial C_m}{\partial\left(\frac{q\,c}{2\,V}\right)} \right|_{q \to 0}$$

INTRODUCTION

The development of the linearized theory of supersonic flow has permitted a first order evaluation of a number of stability derivatives. Second order theories similar to the one introduced by Busemann (ref.1) and extended by Van Dyke (ref.2 and 3) offer possibilities of obtaining second order evaluations of certain stability derivatives, such as lift and moment due to steady pitching, and lift and moment due to constant rate of change of angle of attack. The determination of the damping in roll (in ref. 4) for certain airfoils is an example of the use of a second order theory to obtain stability derivatives.

In this paper a second order theory is developed for two dimensional pitching airfoils at supersonic speeds. This theory yields an expression for the lifting pressure due to steady pitching which enables the stability derivatives C_{L_q} (lift due to pitching) and C_{m_q} (moment due to pitching) to be calculated. The airfoils considered here have arbitrary symmetrical cross sections; however, the analysis can easily be extended to include airfoils with unsymmetrical cross sections.

The partial differential equation for airfoils with a steady pitching velocity is expressible in a form independent of time. Work by Milton D. Van Dyke (ref. 2 and 3) indicates that second order solutions of the partial differential equation of steady supersonic flow can be obtained by iterative methods. The equation considered here is quite similar to, though not the same as, the equation of steady supersonic flow. We shall assume that the second order solution can be obtained by the use of iterative methods.

It will also be assumed that the characteristics are the same for the first and the second order solutions (this assumption was made in ref. 2 and 3). For steady plane flow the second order solution (ref. 2) found by using an iterative method based on the preceding assumption yields the correct second order pressure of the Busemann second order theory. Unfortunately, no such justification of this assumption is known to the authors for the flow associated with pitching airfoils.

ANALYSIS

<u>The Partial Differential Equation:</u> The partial differential equation to be used in the following analysis is a special case of the two dimensional time dependent equation for the potential function of a non-viscous compressible fluid. This equation is (see ref. 4)

$$a^2\left(\Phi_{x_o x_o} + \Phi_{z_o z_o}\right) = \Phi_{tt} + \Phi_{x_o}{}^2 \Phi_{x_o x_o} + 2\Phi_{x_o}\Phi_{z_o}\Phi_{x_o z_o} + \Phi_{z_o}^2\Phi_{z_o z_o} + 2\Phi_{x_o}\Phi_{x_o t} + 2\Phi_{z_o}\Phi_{z_o t} \qquad (1)$$

$$a^2 = c_o^2 - \left[(\gamma-1)\left(\Phi_{x_o}{}^2 + \Phi_{z_o}{}^2 + 2\Phi_t\right)/2\right]$$

This equation is associated with axes fixed in space. For the present problem it is convenient to express the differential equation in terms of a set of axes which are fixed to the airfoil. Since an airfoil with a constant rate of pitch can be considered as flying in a circular flight path with a constant speed, the axes fixed to the airfoil must rotate with respect to the stationary axes with a constant angular velocity q. The relations between the two sets of coordinates, which are illustrated in fig. 1, are

$$x = x_o \cos qt + z_o \sin qt \qquad \text{(a)}$$

$$z = V/q - x_o \sin qt + z_o \cos qt \qquad \text{(b)} \qquad (2)$$

The flow over an airfoil with a constant rate of pitch is steady, thus in the new coordinate system the partial derivatives with respect to time will be zero in the present problem. In the axes attached to the airfoil equation (1) is to the second order

$$-\beta^2 \Phi_{xx} + \Phi_{zz} = \Big[M^2(\gamma - 1)\Phi_x(\Phi_{xx} + \Phi_{zz}) + 2M^2 qx\Phi_{xz} + M^2 q\Phi_z - 2M^2 qz\Phi_{xx} + 2M^2\Phi_x\Phi_{xx} + 2M^2\Phi_z\Phi_{xz}\Big] / V \qquad (3)$$

The Pressure Relation: The time dependent pressure relation is (from ref. 4)

$$C_p = -\Big[2(\Phi_{x_o} + \Phi_t/V)/V\Big] - \left(\Phi_{zo}^2/V^2\right) + \left(\beta^2\Phi_{x_o}^2/V^2\right) + \left(2M^2\Phi_{x_o}\Phi_{t_o}/V^3\right) + \left(M^2\Phi_{t_o}^2/V^4\right)$$

In the axes attached to the airfoil the preceding equation becomes

$$C_p = \left(-2\Phi_x/V\right) + \left(\beta^2\Phi_x^2/V^2\right) - \left(\Phi_z^2/V^2\right) + \left(2qz\Phi_x/V^2\right) - \left(2qx\Phi_z/V^2\right) \qquad (4)$$

Eqs. (3) and (4) will be used in the following analysis.

Solution by Iteration: It will be assumed that eq. (3) can be solved by an iteration procedure. The first order solution is taken as the first approximation to the solution of eq. (3). The first order differential equation is obtained by neglecting the second order terms in eq. (3), and it is given by

$$-\beta^2 \phi_{xx} + \phi_{zz} = 0 \qquad (5)$$

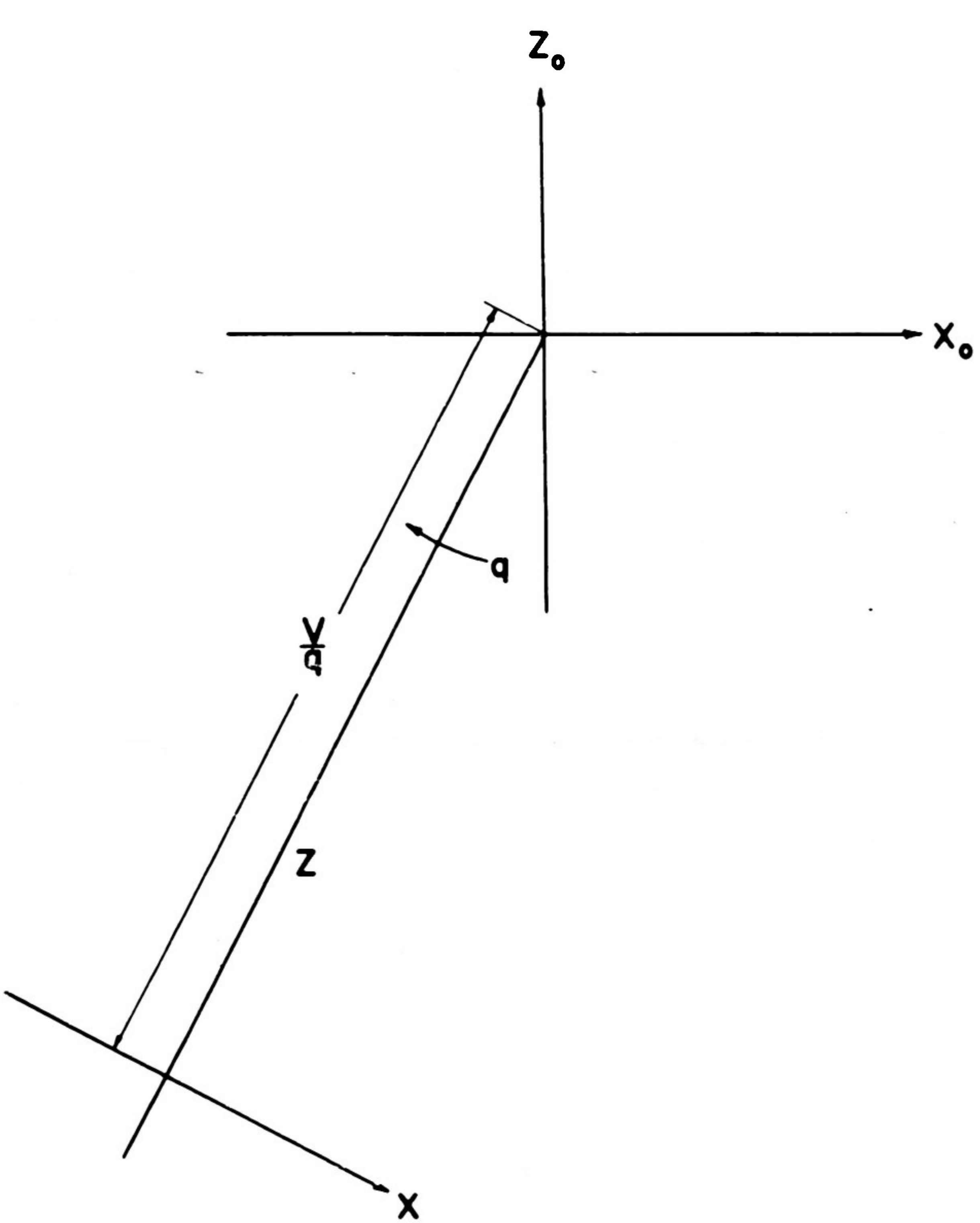


Fig. 1 – Stationary and moving axes.

It is assumed that the second approximation can be found by substituting the first order solution into the right side of eq. (3) and solving the resulting nonhomogeneous equation, which from eq. (3) and (5) is

$$-\beta^2 \psi_{xx} + \psi_{zz} = \Big[M^2(\gamma - 1)\phi_x(\phi_{xx} + \phi_{zz}) + 2M^2 q\, x\, \phi_{xz} + M^2 q\, \phi_z - 2M^2 q\, z\, \phi_{xx} + 2M^2 \phi_x \phi_{xx} + 2M^2 \phi_y \phi_{xy} + 2M^2 \phi_z \phi_{xz}\Big] / V \tag{6}$$

The solution of eq. (6) will be referred to as the second order solution.

Elimination of Terms: It is helpful to investigate the type of solution obtained from eq. (6). The first order equation will be of the form

$$\phi = \alpha\, g_1(x,z) + q\, g_2(x,z) + \epsilon\, g_3(x,z)$$

where ϵ is a thickness parameter and where α, q, and ϵ are small compared to unity. It follows from the preceding expression and eq. (6) that the second order solution will be of the form

$$\psi = \alpha^2 h_1(x,z) + q^2 h_2(x,z) + \epsilon^2 h_3(x,z) + \alpha q h_4(x,z) + \alpha\epsilon h_5(x,z) + q\epsilon h_6(x,z)$$

The terms $\alpha^2 h_1(x,z)$, $\epsilon^2 h_3(x,z)$, and $\alpha\,\epsilon\, h_5(x,z)$ are independent of the rate of pitch; thus they do not contribute to the lifting pressure due to the pitching motion of the airfoil*. Since this paper is concerned only with the lifting pressure due to the pitching, all terms of the forms $\alpha^2 h(x,z)$, $\epsilon^2 h(x,z)$, and $\alpha\epsilon h(x,z)$ can be neglected in the ensuing analysis.

The following argument shows that $q^2 h_2(x, z)$ and $\alpha q h_4(x,z)$ do not contribute to the lifting pressure. The thickness parameter, ϵ, is not present; therefore, the airfoil can be considered a flat plate insofar as these terms are concerned. If the potential of the flow on the upper surface of a flat plate is expressed as

$$\Phi(\alpha, q) = \alpha g_1 + q g_2 + \alpha^2 h_1 + \alpha q h_4 + q^2 h_2$$

the potential on the lower surface is given by

$$\Phi(-\alpha, -q) = -\alpha g_1 - q g_2 + \alpha^2 h_1 + \alpha q h_4 + q^2 h_2$$

the potential difference is

$$\Delta\Phi = 2\,\alpha\, g_1 + 2 q g_2$$

* The terms $\alpha^2 h_1(x, z)$, and $\alpha\epsilon h_5(x, z)$ are associated with steady two dimensional supersonic flow and can be found by use of eq. (46) in ref. (2)

Since for the flat plate the pressure difference between the uppor and lower surface can be found directly from the potential difference the terms $\alpha\, q\, h_4$ and $q^2\, h_2$ do not contribute to the lifting pressure.

The remaining term $q\epsilon h_6\,(x\,,z)$ will be found by use of eq. (6), neglecting all expressions multiplied α^2, $\alpha\epsilon$, q^2, ϵ^2, and αq.

A further consideration of the form of the solution indicates that the second order lifting pressure is linear in the thickness parameter. This can be established by considering a first order solution of the form

$$\phi = q\, g_1(x,z) + \epsilon_1\, g_4\,(x,\, z) + \epsilon_2 g_5\,(x,z)$$

The second order lifting pressure will be of the form

$$\Delta C_p = \left[4\, q\, x/(\beta\, V)\right] + q\, \epsilon_1\, I_1\,(x) + q\epsilon_2\, I_2\,(x)$$

This equation is linear in ϵ_1 and ϵ_2; thus the lifting pressure for various known thickness distributions can be added to obtain the lifting pressure for new thickness distributions. For the two dimensional airfoil this is not of much value for determining analytical solutions, since we shall determine the solution for an arbitrary thickness distribution. This linearity is also true, however, for three dimensional airfoils, and for these it should prove quite useful.

Boundary Conditions: Physical considerations require that the flow be tangent to the surface of the airfoil, and that all velocity perturbations vanish upstream of the airfoil. These boundary conditions may be expressed mathematically as:

$$\left.\begin{aligned}\phi\,(x,\, z) &= 0\\ \psi\,(x,\, z) &= 0\end{aligned}\right\}\qquad \text{upstream of the airfoil's leading edge Mach sheet}$$

and

$$\omega\, .\nabla s = 0$$

where $s(x,z) = 0$ is the equation of the surface of the airfoil.

The equation of the surface of the airfoil may also be expressed as

$$z = \epsilon f(x)$$

Thus

$$\nabla s = -\, i\epsilon\partial f/\partial x + k$$

Since the velocity, ω, can be written as

$$\omega = i(V - zq + \phi_x + \psi_x) + k(qx + \phi_z + \psi_z)$$

it follows that the boundary condition on the body surface is

$$-(V - zq + \phi_x + \psi_x)\,\epsilon f_x + \phi_z + \psi_z + qx = 0$$

Thus the boundary conditions for the first order solution are $\phi(x,z) = 0$ upstream of the airfoil's leading edge Mach sheet, and

$$\phi_z \Big|_{z=0} = -q\,x + V\epsilon\, f_x \tag{8}$$

The boundary conditions for the second order solution are $\psi(x,z) = 0$ upstream of the airfoil's leading edge Mach sheet, and

$$\psi_z \Big|_{z=0} = \phi_x \Big|_{z=0} \epsilon f_x - \epsilon f\, \phi_{zz} \Big|_{z=0} \tag{9}$$

For the airfoils considered here the first order velocity components are discontinuous across the Mach sheet from the leading edge.

To evaluate the effect on the second order solution of the discontinuities in the first order velocity components, we shall assume the Mach sheet from the leading edge to have a small thickness. Within this Mach sheet the first order velocity components will be made continuous so that the discontinuities through it are replaced by continuous functions. This process is illustrated in fig. (2) for the velocity component in the free stream direction. The effect of the discontinuities on the second order solution will be found by obtaining the second order solution within the Mach sheet and then letting the thickness approach zero. This is the same procedure followed in ref. (4) for rolling airfoils.

Only the Mach sheet above the airfoil will be considered since the Mach sheets above and below the airfoil are of the same form. Since they depend only on the initial slope of the airfoil, the discontinuities in the first order velocity components through the leading edge Mach sheet above the airfoil will be the same as those through the Mach sheet above a flat pitching airfoil with a constant angle of attack of the amount $-\epsilon$.

The first order potential function for the flow over the upper surface of a flat pitching airfoil at an angle of attack $-\epsilon$ is given by

$$\phi = \left[q(x - \beta z)^2/2 - V\epsilon(x - \beta z) - qd^2/2 - V\epsilon d \right] / \beta \tag{10}$$

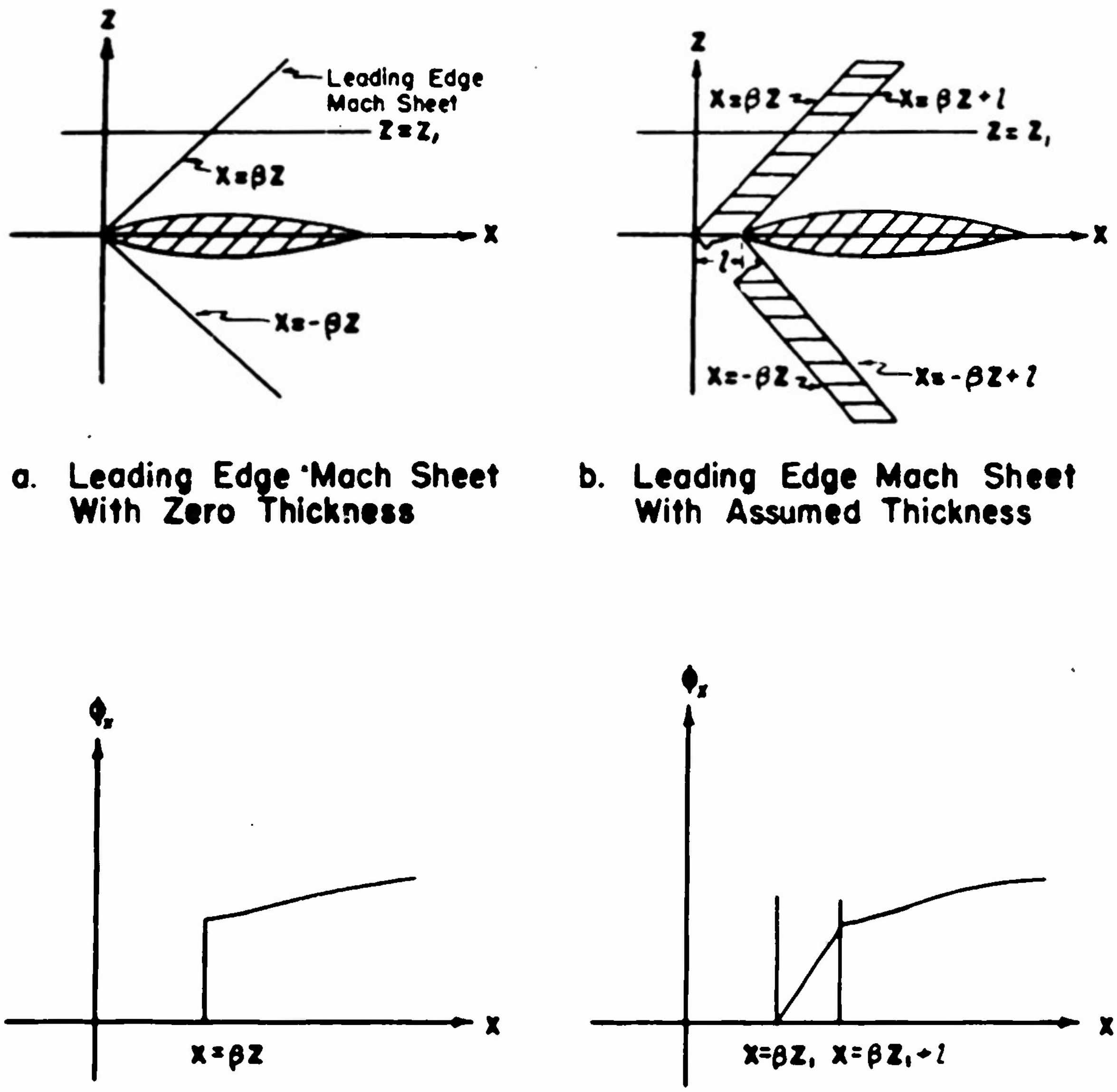
where the airfoil is approximately in the $z = 0$ plane.

The discontinuities in the first order velocity components through the leading edge Mach sheet above the airfoil are from eq. (10).

$$\Delta\phi_x = -qd/\beta - V\epsilon/\beta$$

$$\Delta\phi_z = qd + V\epsilon$$

It will be assumed that the Mach sheet has a small thickness (see fig.2-b).



a. Leading Edge Mach Sheet With Zero Thickness

b. Leading Edge Mach Sheet With Assumed Thickness

c. Discontinuity In ϕ_x Across Leading Edge Mach Sheet Along The Line $Z=Z_1$

d. Plot Of ϕ_x Across Leading Edge Mach Sheet With Assumed Thickness Along The Line $Z=Z_1$

Fig. 2 – An illustration of removing the discontinuity in ϕ_x across the leading edge Mach sheet by assuming the Mach sheet to have thickness.

The velocity components within the sheet will be defined as

$$\phi_x = -(qd + V\epsilon)(x - \beta z + d + \ell)/(\ell \beta)$$

$$\phi_z = (qd + V\epsilon)(x - \beta z + d + \ell)/\ell \qquad (11)$$

where ℓ is the thickness of the Mach sheet in the x direction. Note that within the Mach sheet

$$-\beta^2 \phi_{xx} + \phi_{zz} = 0$$

and that the velocity components are continuous functions in the neighborhood of it.

From eq. (6) and (11) the differential equation within the Mach sheet is

$$-\beta^2 \psi_{xx} + \psi_{zz} = \left[2\, dM^4(\gamma + 1)\, q\epsilon (x - \beta z + d + \ell)/(\beta^2 \ell^2)\right] + \left[2M^2 q\epsilon x/\ell\right] + \left[2 M^2 q\epsilon\beta z/(\beta^2 \ell)\right] + \left[M^2 q\epsilon (x - \beta z + d + \ell)/\ell\right] \qquad (12)$$

It is convenient to express eq. (12) in terms of variables which lie along the characteristics. Let

$$\xi = x - \beta z$$

$$\eta = x + \beta z$$

In terms of the new coordinates (ξ, η) eq. (12) becomes

$$\psi_{\xi\eta} = -\left[2dM^4(\gamma + 1)\, q\epsilon (\xi + d + \ell)/(4 \beta^4 \ell^2)\right] - \left[M^2 q\epsilon (\xi + d + \ell)/(4\ell\beta^2)\right] - \left[M^2 q\epsilon (\xi + \eta)/(4 \beta^2 \ell)\right] - \left[M^2 q\epsilon (\eta - \xi)/(4\beta^4 \ell)\right]$$

This equation can be integrated to yield

$$\psi = -\left[2dM^4(\gamma + 1)\, q\epsilon/(4 \beta^4 \ell^2)\right]\iint (\xi + d + \ell)\, d\xi\, d\eta - \left[M^2 q\epsilon/(4 \beta^2 \ell)\right]\iint (\xi + d + \ell)\, d\xi\, d\eta - \left[M^2 q\epsilon/(4 \beta^2 \ell)\right]\iint (\xi + \eta)\, d\xi\, d\eta - \left[M^2 q\epsilon/(4\beta^4 \ell)\right]\iint (\eta - \xi)\, d\xi\, d\eta$$

The potential on the downstream side of the Mach sheet can be found by evaluating the integrals in the preceding equation and taking the limit as $\ell \to 0$.

The value of the discontinuity in the second order potential function across the leading edge Mach sheet is (from the above equation).

$$\psi\Big|_{x = \beta z - d} = -M^2 q\epsilon f'(-d)\left[M^2 x + (2 + \gamma M^2)d\right] \beta z/(2\beta^4) \qquad (13)$$

Solution of the Partial Differential Equation: The part of the second order potential function which yields the lifting pressure due to the pitching motion will now be determined. The determinations of the potentials over the upper and lower surfaces are similar; therefore, only the flow over the upper surface will be considered in detail.

The first order solution is

$$\phi = \left[q\,(x - \beta z)^2/(2\,\beta)\right] - \left[V\epsilon\, f(x - \beta z)/\,\beta\right] - qd^2/(2\,\beta) \tag{14}$$

It follows from eq. (6) and (14) that the second order potential function must satisfy the nonhomogeneous equation

$$-\beta^2\psi_{xx} + \psi_{zz} = M^2 q\epsilon \left\{ f'(x - \beta z) - \left[M^2(\gamma + 1)/\beta^2\right]\left[(x - \beta z) f''(x-\beta z) + f'(x - \beta z)\right] + 2\,(\beta x + z)\, f''\,(x - \beta z)\,/\,\beta \right\} \tag{15}$$

where the primes denote derivatives with respect to $(x - \beta z)$, and the α^2, αq, $\alpha\epsilon$, and q^2 terms have been neglected. By inspection a particular solution of eq. (15) is found to be

$$\psi_1 = M^2\, q\epsilon \left\{ -\left[z\, f(x - \beta\, z)/(2\,\beta)\right] + \left[M^2(\gamma + 1) z(x - \beta z) f'(x-\beta z)/(2\beta^3)\right] + \left[-2\,x\,\beta z\, f'(x - \beta z) + (\beta^2 - 1) z^2 f'(x-\beta z) + (\beta^2-1) z f(x-\beta z)/\beta\right]/(2\beta^2) \right\} \tag{16}$$

The solution of eq. (15), ψ, consists of a particular integral, ψ_1, plus a complementary function, ψ_2, which must be found so that ψ satisfies the boundary conditions given by eq. (9) and (14). Hence ψ_2 must satisfy

$$-\beta^2\,(\psi_2)_{xx} + (\psi_2)_{zz} = 0$$

and the following boundary conditions:

$$\psi_2\Big|_{x = \beta z - d} = \psi\Big|_{x = \beta z - d} - \psi_1\Big|_{x = \beta z - d} \tag{17}$$

$$\partial\psi_2/\partial z\Big|_{z = 0} = \partial\psi/\partial z\Big|_{z = 0} - \partial\psi_1/\partial z\Big|_{z = 0}$$

$$= \left[q\epsilon/(2\beta^3)\right]\left\{-M^4\gamma + (\beta^4 + 2\beta^2 - 1) x f'(x) + (1 + \beta^2 - 2\beta^4) f(x)\right\} \tag{18}$$

By inspection the complementary function is found to be

$$\psi_2 = \left[q\epsilon/(2\beta^4)\right]\left\{\left[M^4\gamma - (\beta^4 + 2\beta^2 - 1)\right](x - \beta z) f(x-\beta z) + (-M^4\gamma + 3\,\beta^4 + \beta^2 - 2)\int_{-d}^{x-\beta z} f(\lambda)\, d\lambda\right\} \tag{19}$$

The desired solution is

$$\psi = \psi_1 + \psi_2 = \left[q\epsilon/(2\beta^4)\right]\Big\{ M^2(M^2\gamma + 1 - \beta^2)\,\beta\, x\, z\, f'(x - \beta z) - M^2(M^2\gamma - 2)\,\beta^2 z^2 f(x - \beta z) + \left[-M^2\beta z + M^4\gamma(x - \beta z) - (\beta^4 + 2\beta^2 - 1)(x - \beta z)\right] f(x - \beta z) + \left[-M^4\gamma + 3\beta^4 + \beta^2 - 2\right]\int_{-d}^{x-\beta z} f(\lambda)\,d\lambda\Big\} \quad (20)$$

Eq. (20) is the part of the second order potential function which yields the entire second order contribution to the lifting pressure due to pitching.

Lifting Pressure: The lifting pressure distribution can be expressed as

$$C_p = -C_p\Big|_{\substack{\text{upper}\\ \text{surface}}} + C_p\Big|_{\substack{\text{lower}\\ \text{surface}}}$$

From eq. (4) and (20) it follows that for symmetrical airfoils the lifting pressure distribution is

$$\Delta c_p = -\left[4q/(\beta V)\right]\left[x + \left\{\epsilon/(2\beta^3)\right\}\left\{(M^4\gamma + \beta^4 - 2\beta^2 + 1)\,x f'(x) - M^2 f(x)\right\}\right] \quad (21)$$

Fig. (3) presents the lifting pressure distribution on a ten per cent thick wedge pitching about the c/2 point for various Mach numbers.

Fig. (4) presents the pressure distribution on a ten per cent wedge at Mach number 1.5 for various positions of the axis of pitch.

Fig. (5) presents the lifting pressure distribution on a five per cent thick airfoil with a parabolic cross section pitching about the c/2 point for various Mach numbers.

Fig. (6) presents the lifting pressure distribution on a five per cent thick airfoil with a parabolic cross section at Mach number 1.5 for various positions of the axis of pitch.

Stability Derivatives: The effect of thickness on the lift and moment, C_{L_q} and C_{m_q}, due to steady pitching can be found by use of eq. (21). The stability derivatives C_{L_q} and C_{m_q} can be expressed as

$$C_{L_q} = \left[\partial C_L/\partial\left\{qc/(2V)\right\}\right]_{q\to 0} = \left[1/\left\{qc^2/(2V)\right\}\right]\int_{-d}^{c-d} \Delta c_p\, dx$$

$$C_{m_q} = \left[\partial C_m/\partial\left\{qc/(2V)\right\}\right]_{q\to 0} = -\left[1/\left\{qc^3/(2V)\right\}\right]\int_{-d}^{c-d} x\,\Delta c_p\, dx$$

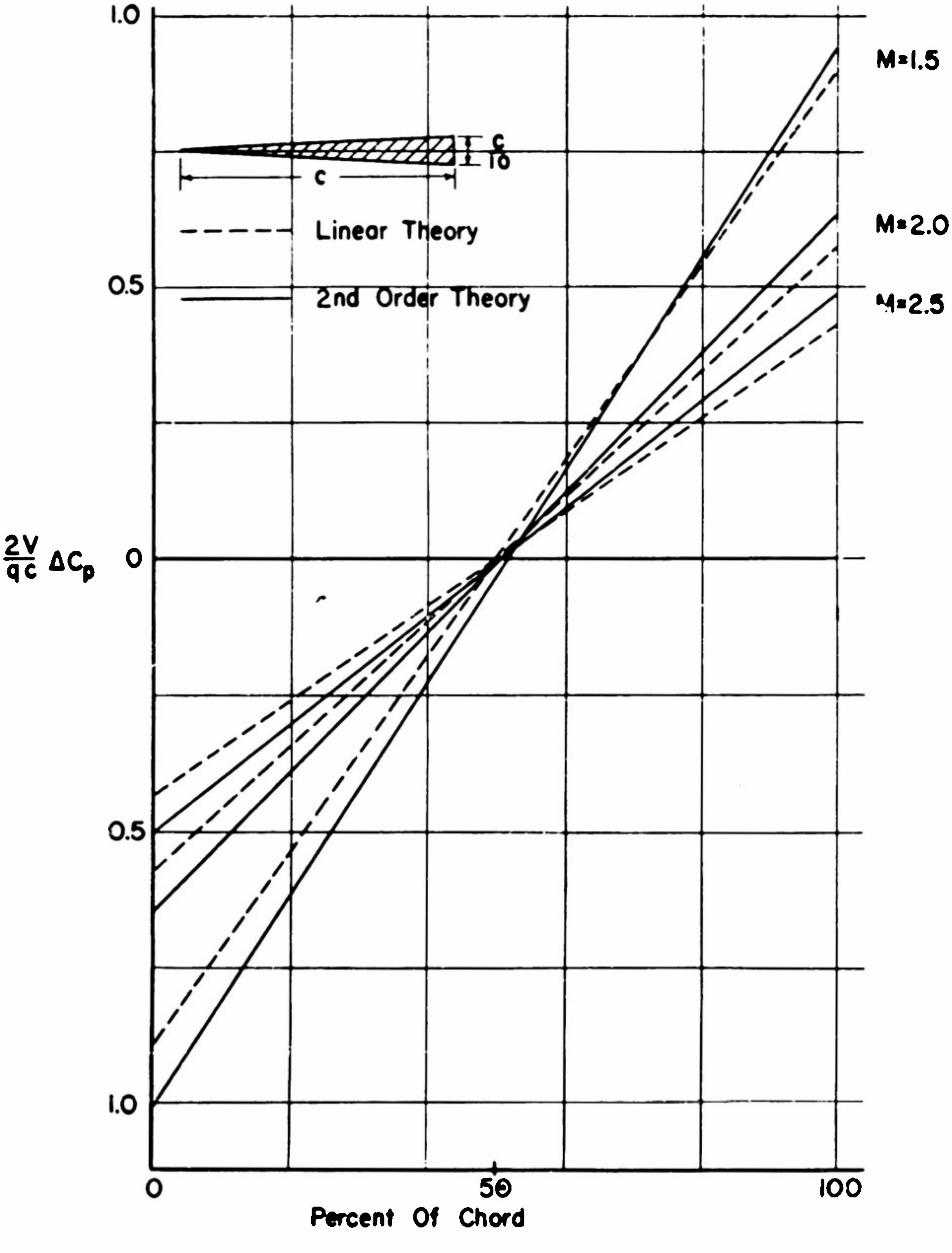


Fig. 3 – The lifting pressure distribution on a ten percent thick wedge pitching about the c/2 point for various Mach numbers.

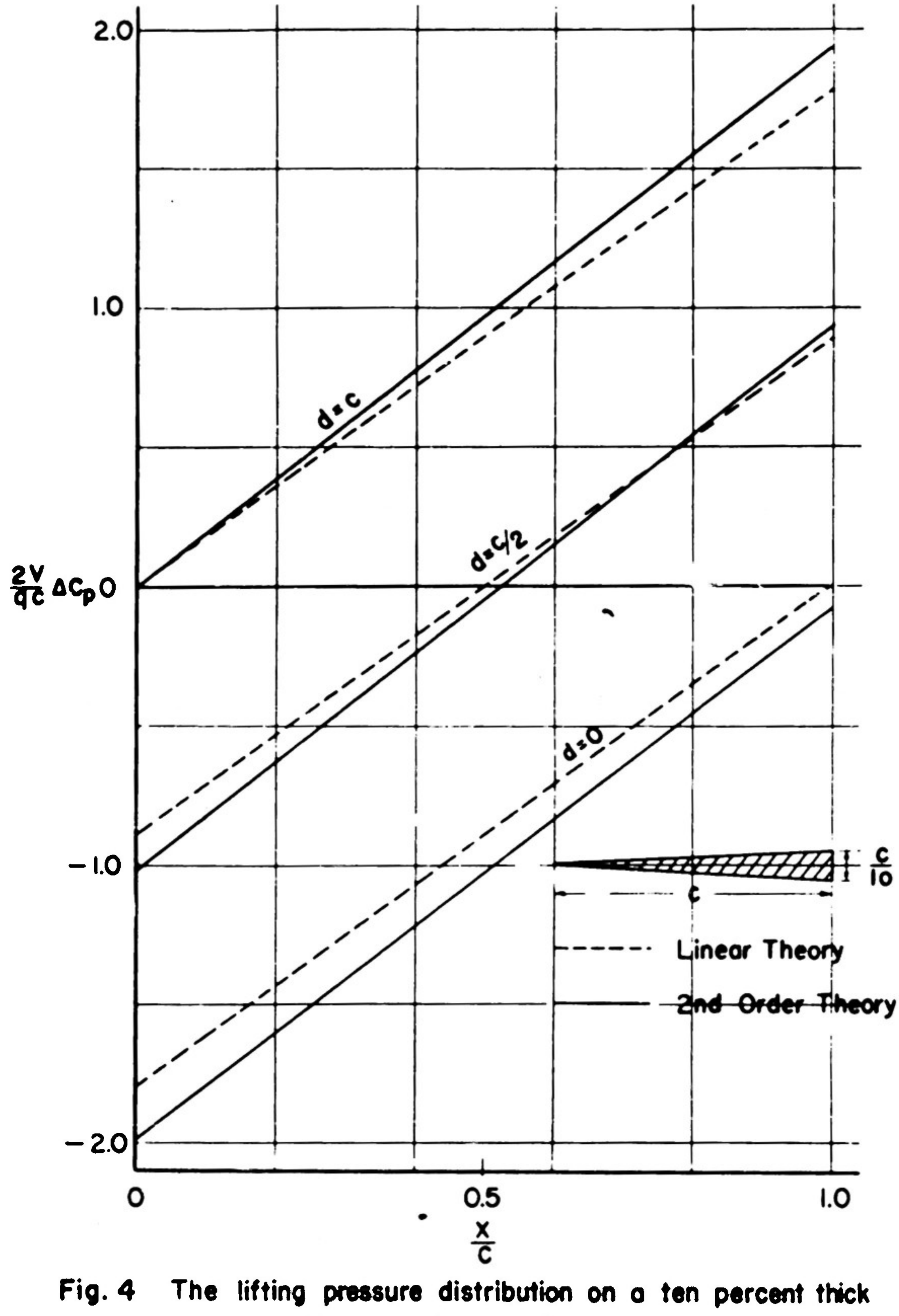


Fig. 4 The lifting pressure distribution on a ten percent thick wedge at Mach number 1.5 for various positions of the axis of pitch.

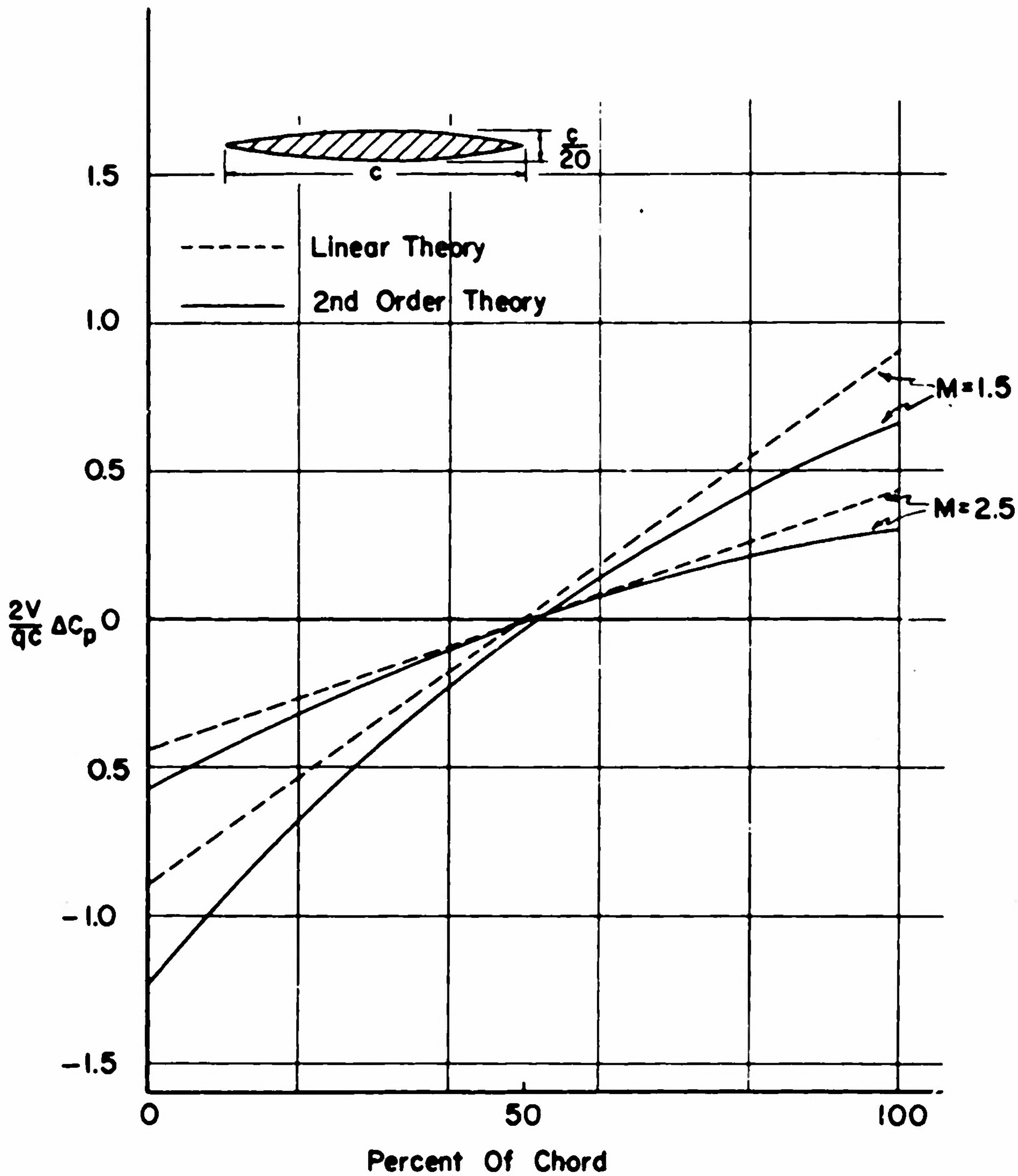


Fig. 5 The lifting pressure distribution on a five percent thick airfoil with parabolic cross section pitching about the c/2 point for Mach numbers 1.5 and 2.5 .

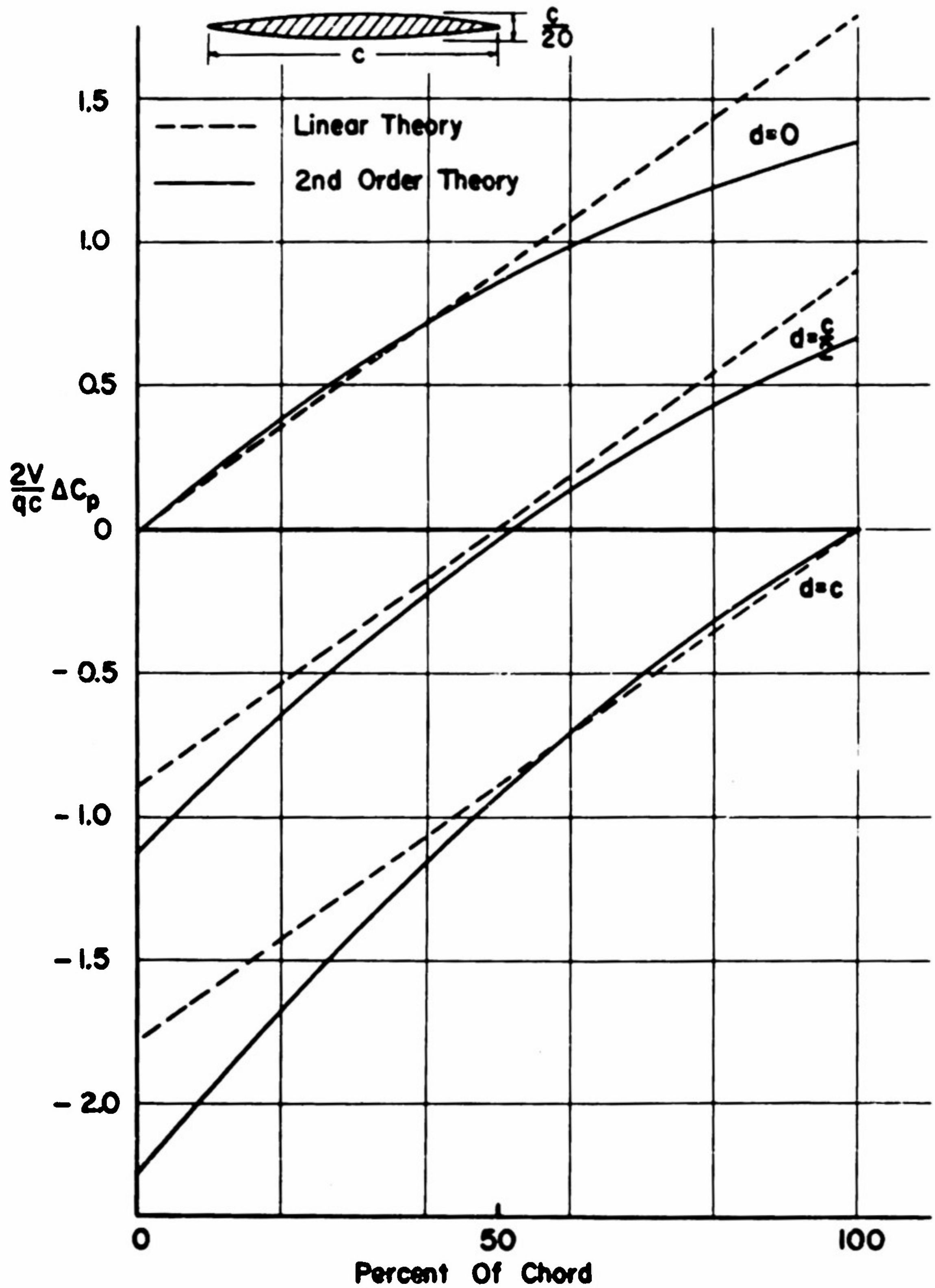


Fig 6 The lifting pressure distribution on a five percent thick airfoil with parabolic cross section at Mach number 1.5 for various positions of the axis of pitch.

From the preceding relations and eq. (21) the C_{L_q} and the C_{m_q} of an airfoil are

$$C_{L_q} = 8d/(\beta c) - 4/\beta - \left[4\epsilon/\beta^4\right]\Big\{\left[M^4\gamma + (\beta^2-1)^2\right]\left[(1-d/c)/c\right] f_{TE} - \left[M^4\gamma + (\beta^2-1)^2 + M^2\right]\int_{-d}^{c-d}\left[f(\xi)/c^2\right] d\xi\Big\} \tag{22}$$

and

$$C_{m_q} = -\left[8/(3\beta)\right]\left[1 - 3\,d/c + 3\,d^2/c^2\right] - (4\epsilon/\beta^4)\Big\{\left[M^4\gamma + (\beta^2-1)^2\right]\left[(1-d/c)^2/c\right] f_{TE} - \left[2M^4\gamma + 2(\beta^2-1)^2 + M^2\right]\int_{-d}^{c-d}\left[\xi f(\xi)/c^3\right] d\xi\Big\} \tag{23}$$

Eq. (23) indicates that the effect of thickness on the C_{m_q} is zero for an airfoil pitching about the c/2 point with a thickness distribution which is chord-wise symmetrical with respect to the mid-chord point.

Fig. 7 through 10 present the variation of the C_{L_q} for a ten per cent thick wedge and a five per cent parabolic airfoil with Mach number and the position of the axis of pitch. Fig. 7 and 9 indicate that for $d = 0$ the thickness decreases the C_{L_q} of a wedge airfoil and increases the C_{L_q} for a parabolic airfoil.

Fig. 11 through 14 present the variation of the C_{m_q} for a ten per cent thick wedge and a five per cent parabolic airfoil with Mach number and the position of the axis of pitch. Fig. 11 and 13 indicate that for $d = 0$ the thickness decreases the C_{m_q} for a wedge airfoil and increases the C_{m_q} for a parabolic airfoil.

CONCLUDING REMARKS

The airfoils considered in this paper have symmetrical thickness distributions. But since the flow over the upper and lower surfaces of the airfoils treated here are independent of each other, the aerodynamic properties due to pitching of airfoils with unsymmetrical thickness distributions can easily be determined from the results obtained here.

The limitations of the Busemann second order theory have been investigated (see ref. 5) Since the theory contained in the present paper is closely associated with the Busemann second order theory, it seems likely that the results presented herein have similar limitations.

John C. Martin

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Nathan Gerber

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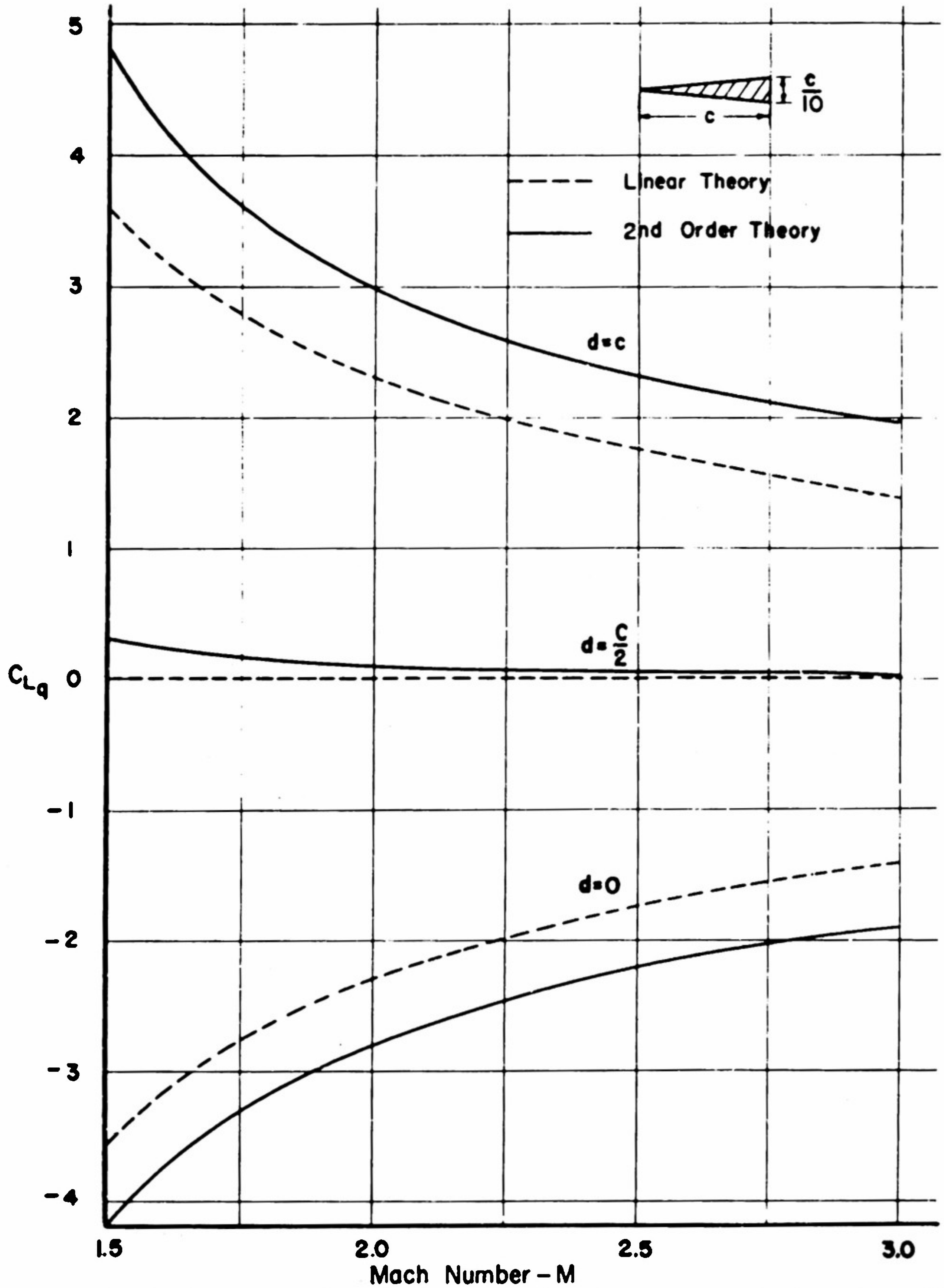

Fig 7 The variation of the C_{L_q} of a ten percent thick wedge with Mach number for various positions of the axis of pitch.

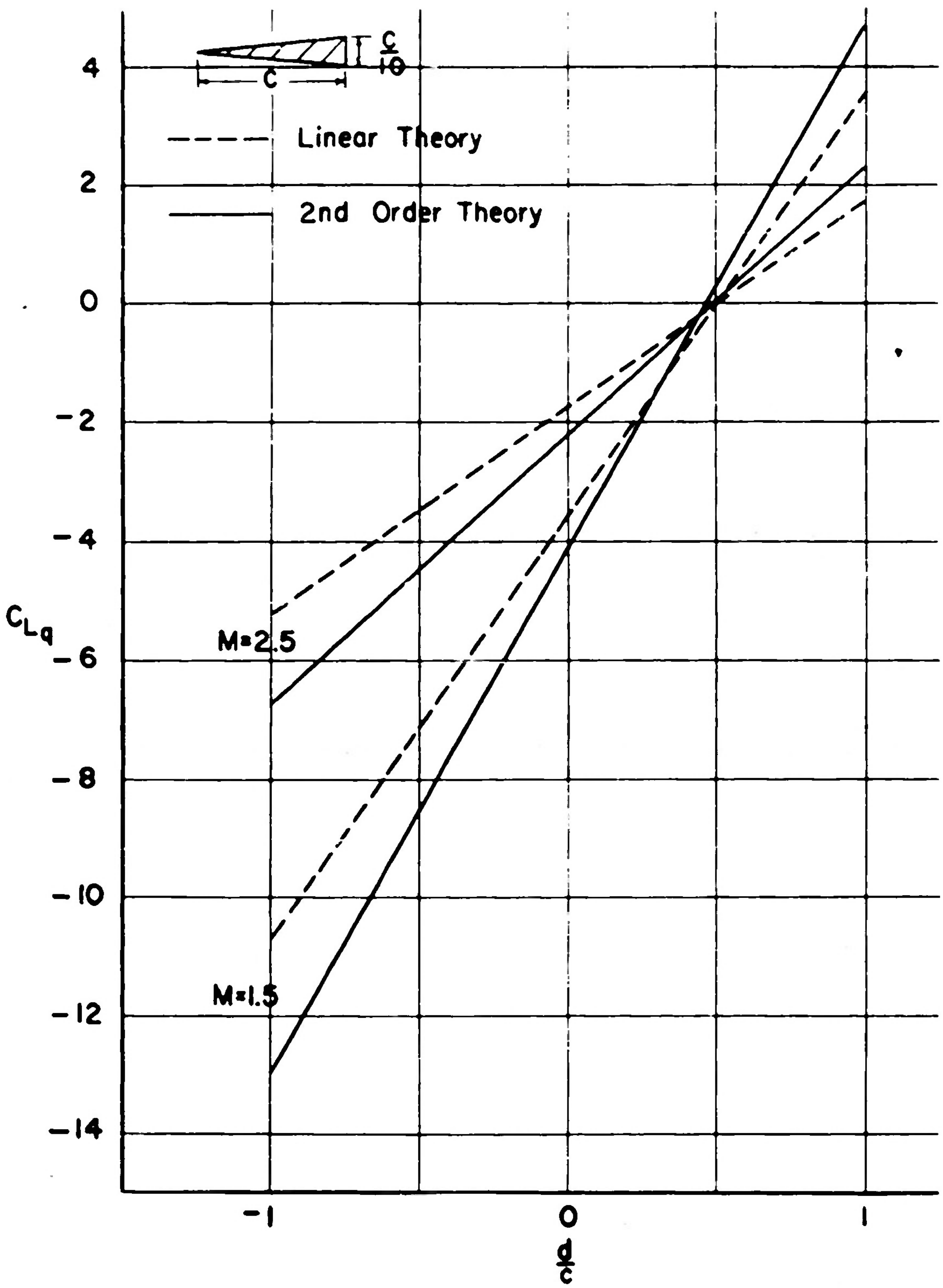

Fig 8 The variation of the C_{L_q} of a ten percent thick wedge with the location of the axis of pitch for Mach numbers 1.5 and 2.5.

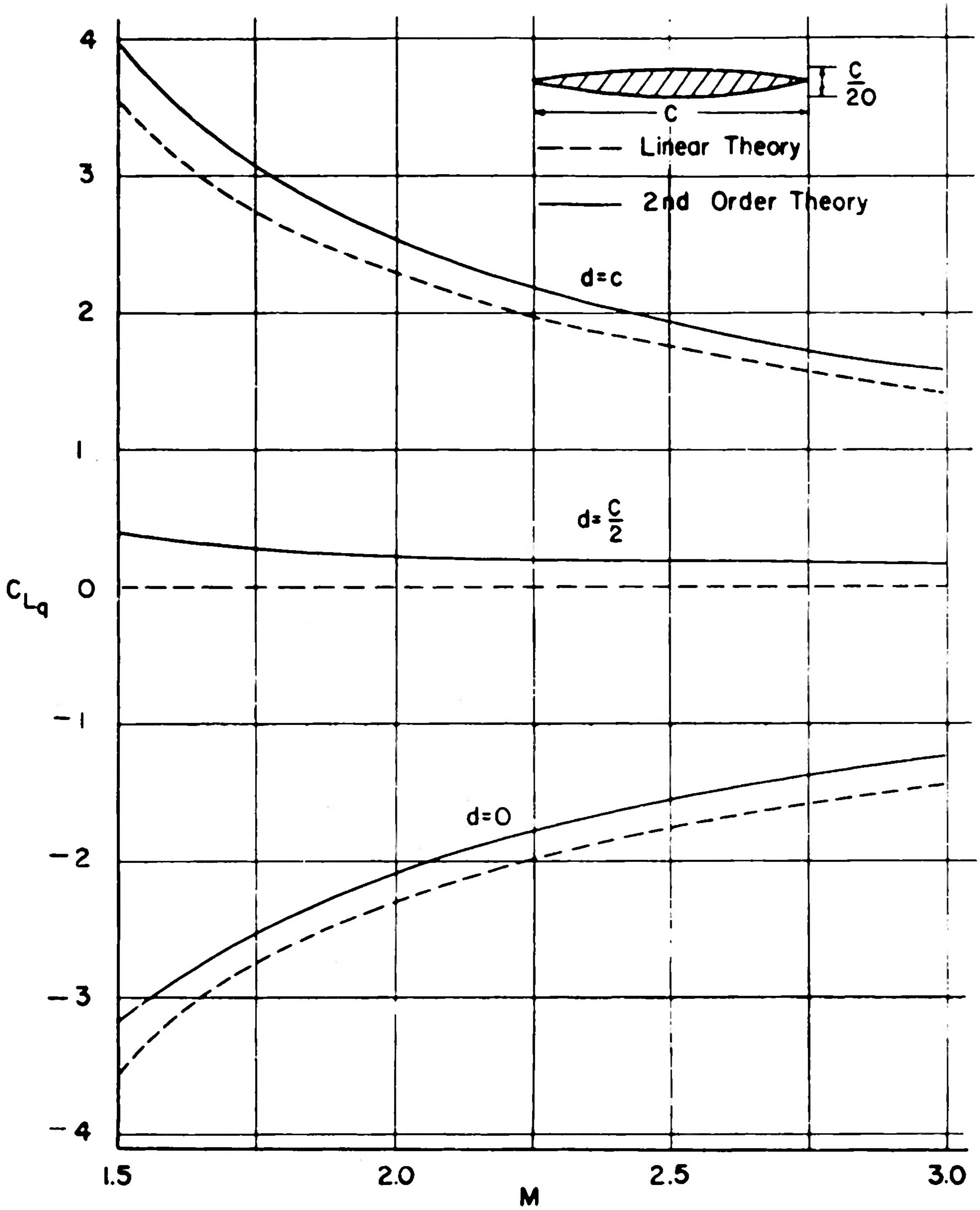

Fig 9. The variation of the C_{L_q} of a five percent thick parabolic airfoil with Mach number for various positions of the axis of pitch.

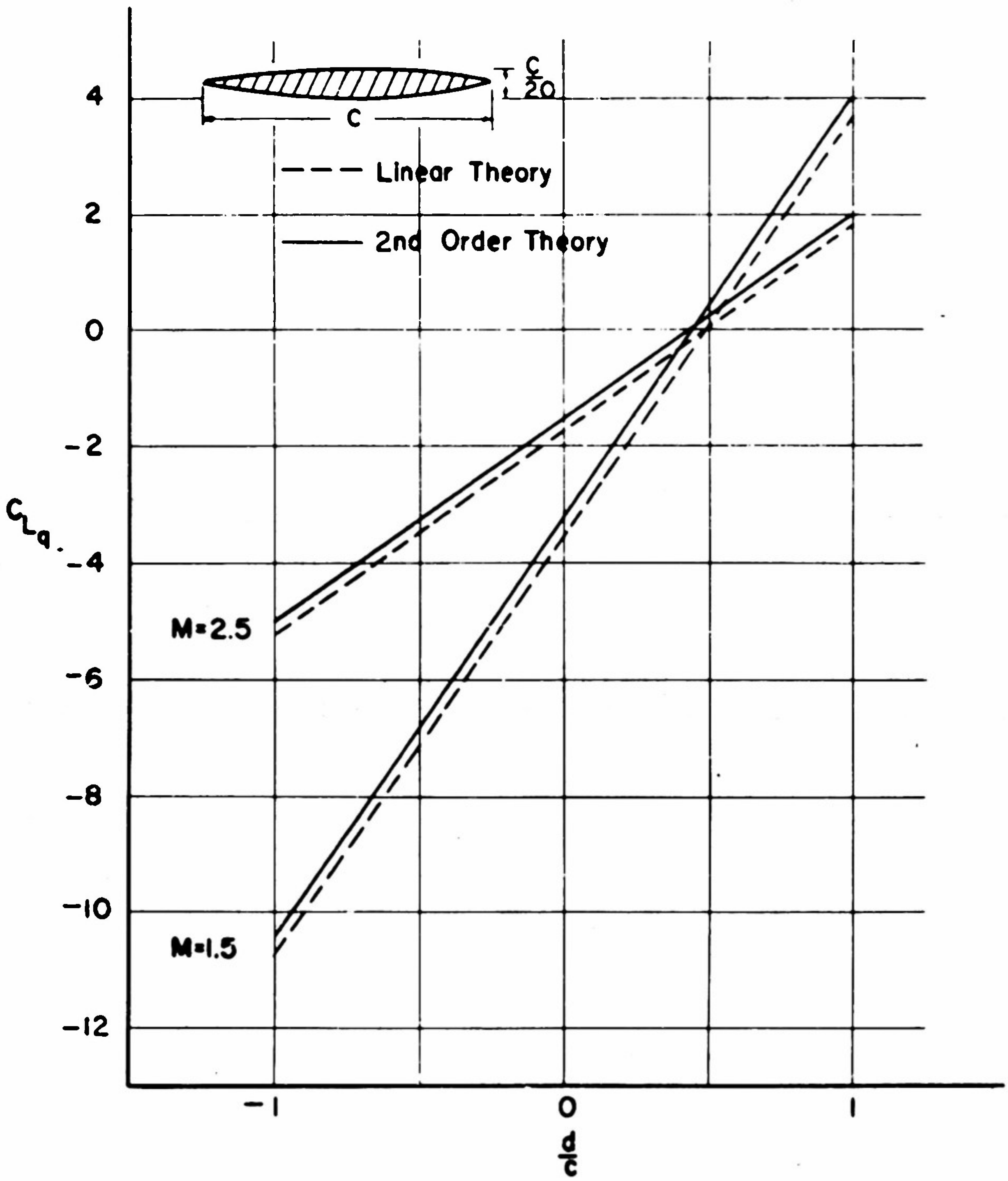

Fig 10 The variation of the $C_{L_{\dot q}}$ of a five percent thick parabolic airfoil with the location of the axis of pitch for Mach numbers 1.5 and 2.5.

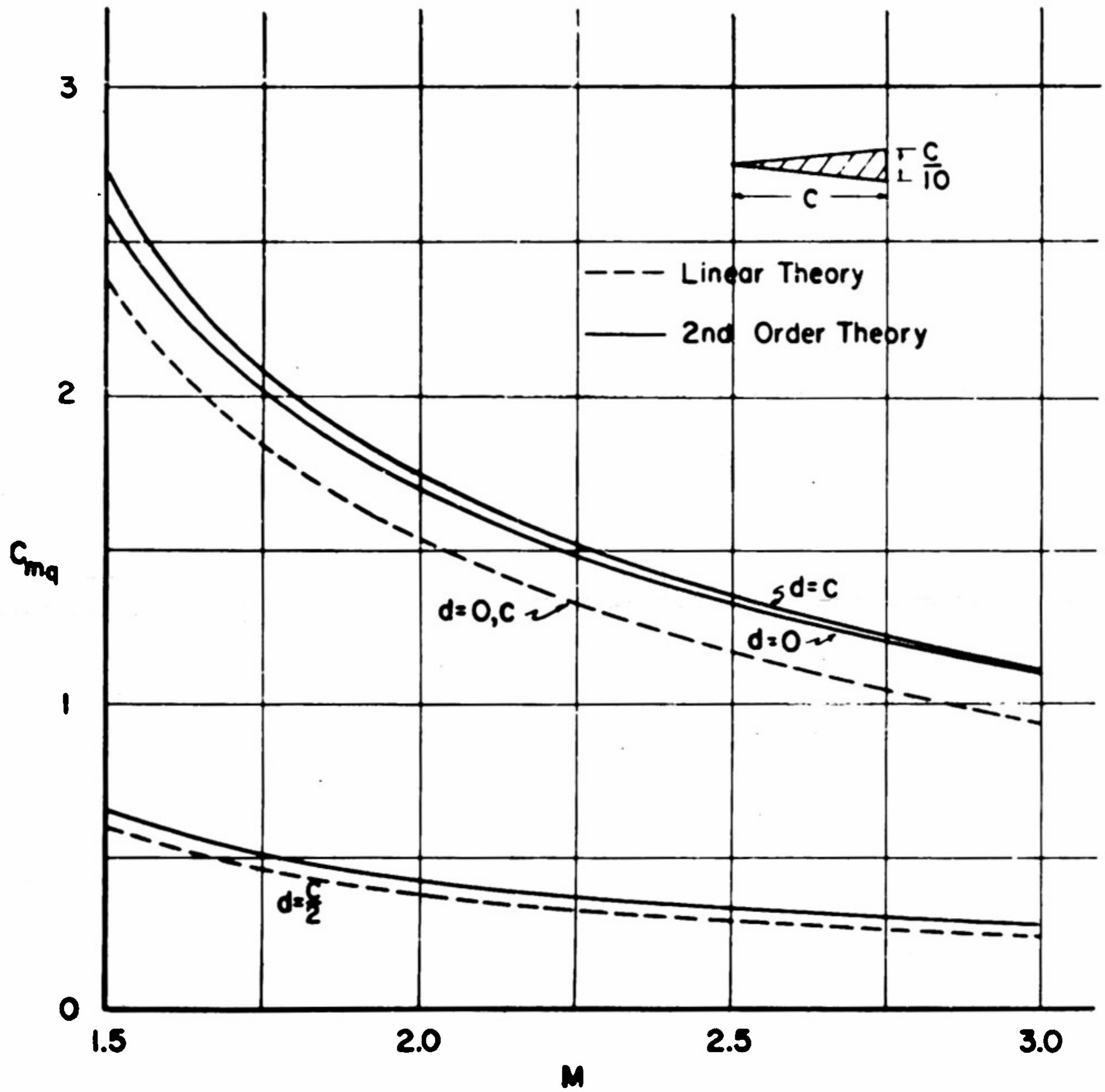

Fig 11 The variation of the C_{m_q} of a ten percent thick wedge airfoil with Mach number for various positions of the axis of pitch.

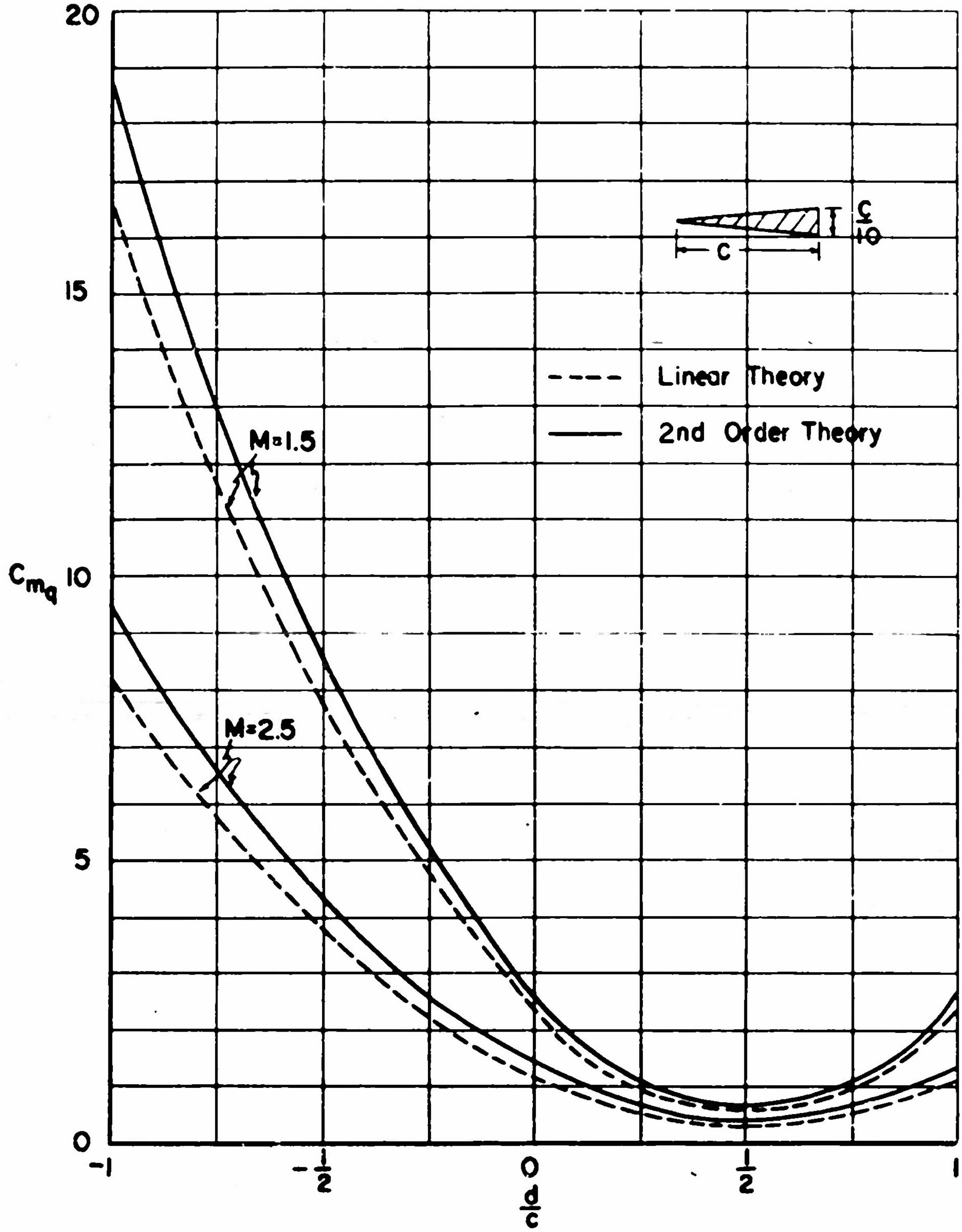

Fig 12 The variation of the C_{m_q} of a ten percent thick wedge airfoil with the location of the axis of pitch for Mach numbers 1.5 and 2.5.

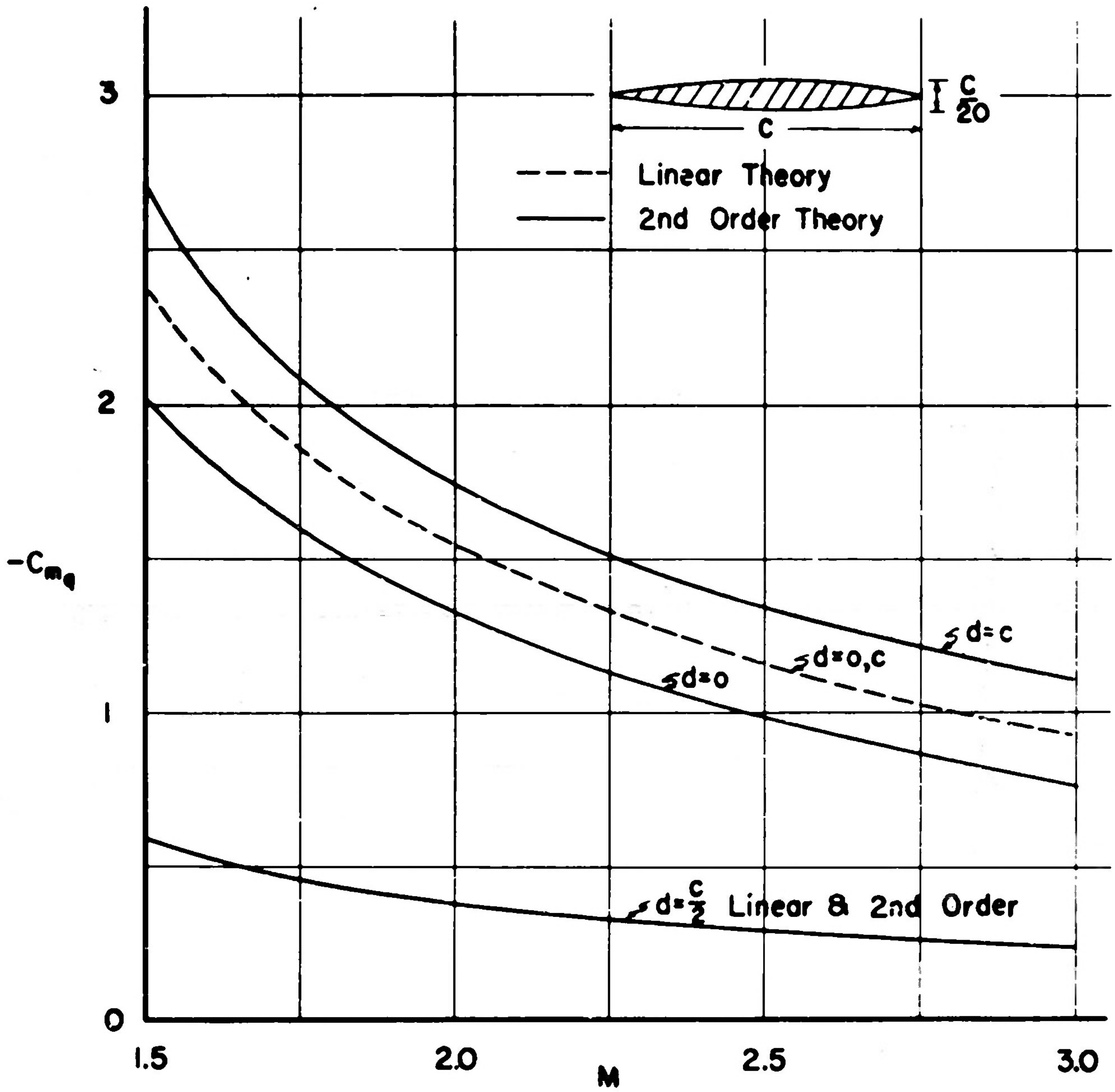

Fig. 13. The variation of the C_{m_q} of a five percent thick parabolic airfoil with Mach number for various positions of the axis of pitch.

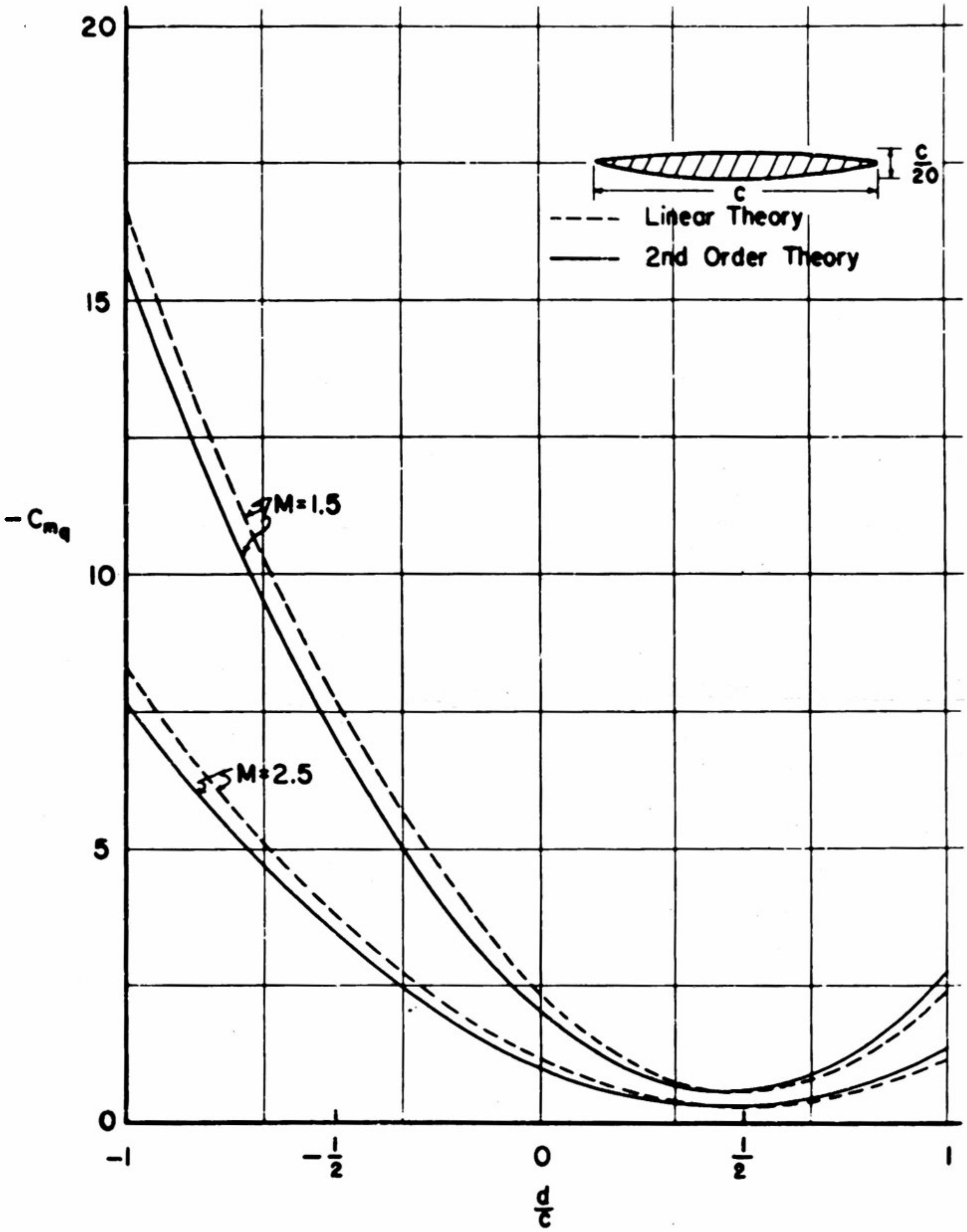

FIG. 14. The variation of the C_{m_q} of a five percent thick parabolic airfoil with the location of the axis of pitch for Mach numbers 1.5 and 2.5.

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